

Department of Mathematics
Brooklyn College
Final Examination: Fall 2016
Mathematics 1401

INSTRUCTIONS: Answer any TEN questions. Each problem is worth 10 points. All work must be shown for full credit.

1. Let U be a set 50 people. Let A be those in the group over the age of 25; B be the set of those in the group who are 18 or older; and C be those in the group between the ages of 20 and 30 inclusive.
 - (a) Draw a Venn diagram showing sets A , B , and C that accurately reflects the relationship between the sets.
 - (b) Describe in words:
 - i. The set $A \cap C$.
 - ii. The set \overline{B} .
 - iii. The set $A \cap \overline{B}$.
 - (c) Suppose there are THREE people under the age of 18; SEVEN people who are 18 or 19; TWENTY-EIGHT people who are over the age of 25; and SIXTEEN people who are between 20 and 30 inclusive.
 - i. How many people are older than 30?
 - ii. How many people are between 20 and 25 (inclusive)?
 - iii. Find the probability that a person picked at random from the group will be older than 25 but 30 years or younger?
2. Perform the following operations.
 - (a) Subtract: $103_{\text{six}} - 34_{\text{six}}$.
 - (b) Add: $77_{\text{eight}} + 44_{\text{eight}} + 212_{\text{eight}}$.
 - (c) Express the number of \square shown below as a number in base THREE
 $\square\square\square\square\square\square\square\square\square\square\square\square\square\square\square\square$
 - (d) Express as a number in base-ten: 132_{four}

3. Answer the following questions.

- (a) Convert into a common fraction: $0.\overline{405}$. Reduce your answer to lowest terms.
- (b) Write the number described in words as a reduced common fraction **AND** as a percentage: thirty-six thousandths.
- (c) Suppose pencils come in packages of 18 and erasers that fit on top of these pencils come in packages of 24 (Assume that you cannot buy partial packages).
 - i. What is the **smallest** number of pencils and erasers you should buy so that each of 200 students will be provided with one pencil and one matching eraser?
 - ii. How many packages of pencils will you need and how many packages of erasers?
 - iii. How many pencils and how many erasers will be left over?

4. Answer the following.

- (a) Prove from the relevant definitions: $3 \times 5 = 15$.
- (b) Let $N = 3^{11}5^{37}7^{13}$. Without multiplying and dividing, determine if:
 - i. N is divisible by 75. Fully explain your answer.
 - ii. N is divisible by 140. Fully explain your answer.
- (c) Suppose N is an even number, and $5N + 11$ and $N + 1$ are **NOT** relatively prime. Find the GCD of $5N + 11$ and $N + 1$.

5. Answer the following questions. Give an exact answer using a mixed number or common fraction.

- (a) Maria poured $3\frac{1}{4}$ cups of water into a container and noticed that the water filled one-third of the container. How many cups of water will be required to fill the whole container?
- (b) A recipe calls for $3\frac{1}{4}$ cups of flour. You want to make one-third of the recipe. How much flour do you need?
- (c) Which problem can be solved by calculating $3\frac{1}{4} \div 3$?

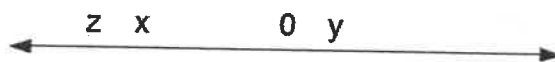
6. A teacher gave a quiz to five students. The maximum possible grade was 10.
- If possible, give a set of five quiz scores with mean 8 and median 5; if not possible, explain why not.
 - If possible, give a set of five quiz scores with mean 5 and median 8; if not possible, explain why not.
 - Suppose the median grade is 7. What is the *highest* possible mean for the five quiz scores?
 - Suppose the mean grade is 8. What is the *lowest* possible median for the five quiz scores?

7. Answer the following questions.

- Place parentheses as needed to make the expression give the least possible value (which might be a fraction or negative).

$$8 \div 2 \times 4 + 6 - 2$$

- A numberline is shown, with the locations of the numbers x , y , and z as shown; you may assume the placement is accurate.



In **EACH** of the following, determine which is greater.

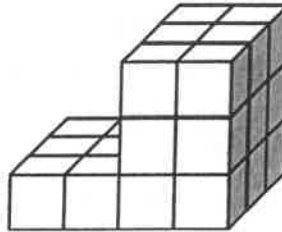
- x or z
- $|x|$ or $-z$
- $|x|$ or $|y|$
- y or $y - x$
- $x + z$ or $y + x$

8. Answer the following.

- Solve: $|x - 1| = 3$
- A girl has 54 inches of cloth to make doll costumes. Each costume requires $4\frac{2}{3}$ inches of cloth. How many costumes can she make, and how much material will be left over? Give an exact answer in inches using a **mixed number**.

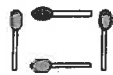
(c) Prove from the definitions: $11 + (-3) = 8$.

9. Assume the figure shown is made out of identical cubes. Each cube has sides with a length of 1 cm.



- Find the surface area of the solid.
- Find the volume of the solid.
- Suppose the figure is broken apart into individual cubes (each with a side length of 1 cm).
 - What will the **total** volume of all the cubes be?
 - What will the **total** surface area of all the cubes be?

10. Assume that the pattern continues for the following sequence of matchstick figures:



1st Figure



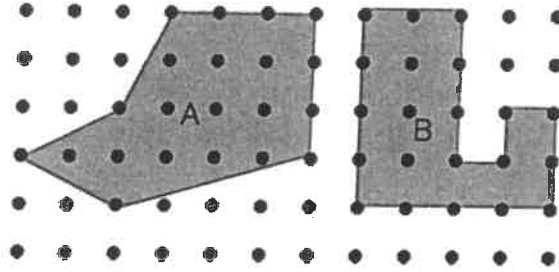
2nd Figure



3rd Figure

- How many matchsticks will there be in the 4th and 5th figure?
- How many matchsticks will there be in the n th figure?
- How many matchsticks will there be in the 100th figure?
- Is there a figure in the sequence that is made up of exactly 78 matchsticks? If so, which one? If not, why not?
- You have a box that contains 200 matchsticks. How many of the figures in the sequence can you build from this box? (Assume that once you've used the matchsticks to build one of the figures, you can't re-use the matchsticks, and that you must build the figures in order)

11. Consider the following figure. Assume the points are 1 unit apart horizontally, and 1 unit apart vertically.



- (a) Find the area of each figure.
(b) Find the perimeter of Figure B.

